JOURNAL OF
EMERGING INVESTIGATORS

Article

The impact of effective density and compressive strength on the structure of crumpled paper balls

Hayley Chu, Rodolfo Fieller
Yew Chung International School of Shanghai Century Park, Shanghai, China

SUMMARY

Crumpling is the process whereby a sheet of paper undergoes deformation to yield a three-dimensional structure comprising a random network of ridges and facets with variable density. A regular sheet of paper can be easily teared and is very flimsy, yet when a sheet of paper is crumpled into a ball, the crumpled paper becomes much sturdier and has a large compressive strength, which is a material's maximum compressive load divided by its cross-sectional area. Scientists have extensively studied this phenomenon due to its peculiar structure; however, the physics behind paper crumpling is not yet completely understood. In this study, we investigated the structure of crumpled paper in two ways. In the first part, we explored the effective density of crumpled paper balls and hypothesized the diameter cubed of a paper ball crumpled from a square paper sheet is directly proportional to the side length squared of the paper sheet. In the second part, we studied the relationship between the number of times an A4 piece of paper is crumpled and its compressive strength. We hypothesized that the more times a paper sheet is crumpled, the greater its compressive strength. Our results supported both of our hypotheses: The first experiment demonstrated there is a directly proportional relationship between the diameter cubed of the ball and the side length squared of the sheet. We discovered in the second experiment that there was a relatively strong linear relationship between the number of times a paper sheet is crumpled and its compressive strength.

INTRODUCTION

When one crushes a sheet of paper and throws it into the bin, perhaps out of frustration, they may not realize the strange phenomenon that results from this seemingly mundane action. Crumpling is the process whereby a sheet of paper undergoes deformation to yield a three-dimensional structure comprising a random network of ridges and facets with variable density (1). A regular sheet of paper can be easily torn and is very flimsy, yet when a sheet of paper is crumpled into a ball, the crumpled paper becomes much sturdier and has a large compressive strength, which is a material's maximum compressive load divided by its cross-sectional area. A crumpled paper ball is estimated to contain around 75-90% air and cannot easily be compacted further (2).

The phenomenon whereby a sheet of paper becomes much stronger after being crumpled has been studied extensively by scientists due to its peculiar structure. Some have used methods ranging from "kvetching" (3), which refers to crushing material in a cylindrical container, to X-ray microtomography, an imaging technique that assembles three-dimensional images from thousands of two-dimensional, cross-section snapshots (4). However, the physics behind paper crumpling is not yet completely understood by scientists. Thus, this study aims to explore why crumpled paper has such a complicated and strong structure.

Bansal, Chowdhry, and Gyaneshwaran (5) studied the relationship between the side length of square sheets of paper that were crumpled into paper balls and the diameter of the crumpled balls, thereby calculating the effective density of the crumpled paper balls. Our first investigation attempted to replicate their experiment.

We have yet to come across a paper investigating the relationship between the number of times a piece of paper is crumpled and its compressive strength. We investigated this relationship by first crumpling an A4 sheet of paper a different number of times then applying compression force by hand to see how much compressive load is needed to cause significant deformation due to crushing.

Crumpled sheets are made of four main structures, which include the bend, the fold, the developable cone (d-cone) and the stretching ridge (**Figure 1**). Bending occurs when the two ends of a sheet of paper are placed on top of another so there is a bend in the middle. When force is applied to the bend, the paper will be folded, creating a fold. When stretching is

The four main structures of crumpled sheets

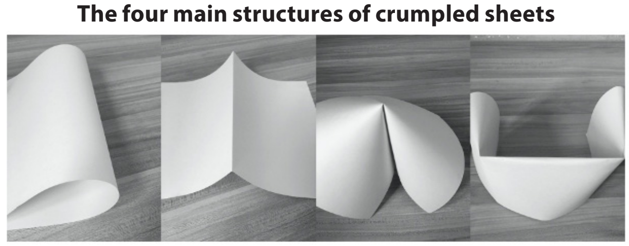

Figure 1. From left to right: the bend, the fold, the d-cone and the stretching ridge.

concentrated at one point and the rest of the sheet is bending in a cone-like shape, the vertex of the sheet is called a d-cone (6). When two d-cones are created in a sheet, stretching will occur between the d-cones and form a stretching ridge. Croll *et al*. discovered through laser scanning confocal microscopy of hand-crumpled paper that energy stored in a ridge is thought to be stored mostly along the ridge peak, seen from the two corners of the stretching ridge at the rightmost frame from **Figure 1** (2). They also concluded that, in the stretched ridges, the d-cones are fixed. While compression attempts to push them together, the d-cones cannot move and only bending occurs in the structure before the ridge buckles. This shows that the ridges deliver energy into the available soft modes (bending), which progressively stiffen until the ridge structure collapses. Stretching ridges have a high buckling strength, which measures how much force a material can withstand before buckling (2). Therefore, bending in the stretching ridge is responsible for the strength of both elastic and plastic crumples. Thus, we hypothesized that the more times a paper sheet is crumpled, the greater its compressive strength.

The first experiment showed the effective density remained almost constant and there is a directly proportional relationship between the diameter cubed of the ball and the side length squared of the sheet. We discovered in the second experiment that there was a relatively strong linear relationship between the number of times a paper sheet is crumpled and its compressive strength.

RESULTS

In order to investigate the relationship between the side length of a square paper sheet and the diameter of the crumpled paper ball for the first experiment, paper sheets of varying side lengths were crumpled into balls, the diameters were measured for each ball. This was repeated twice for each side length for a total of three trials (**Table 1**). For example, the diameter of the paper ball from the sheet of 5 cm side length was measured to be 1.05 cm in Trial 1 and the diameter cubed was 1.16 $cm^{3.}$

The standard dimensions of A4 paper are 21 cm x 29.7 cm x 0.01 cm, and the standard mass is 4.50 g. Thus, the measured density for an A4 sheet of paper is:

$$\rho_p = \frac{mass}{volume} = \frac{4.5\ g}{21\ cm \times 29.7\ cm \times 0.01\ cm} = 0.72\ g/cm^3 \tag{6}$$

The effective density for the paper with a side length of 5 cm was calculated to be:

$$\rho_e = \frac{6\rho_p l^2 t}{\pi d^3} = 0.30 \pm 0.03\ g/cm^3 \tag{7}$$

The average effective density for the paper of all side lengths (on the very right column) for all three trials was then calculated to be 0.24±0.03 g/cm^3. This value is significant as it remains mostly constant for all side lengths and determines the diameter cubed of each crumpled paper ball for each side length squared of the square sheet.

The margin of error for side length shown in **Table 1** was derived from the systematic error from the ruler, which was half of the measuring unit. As the measuring unit was 0.1 cm, the margin of error was ±0.05 cm. The margin of error for the side length squared was calculated by multiplying the margin of error of the side lengths by two to give ±0.01 cm^2. The margin of error for the diameter was derived from the systematic error from the Vernier caliper, with the measuring unit of 0.01 cm, thus giving the margin of error of ±0.005 cm. For the diameter cubed, the margin of error was the margin of error of the diameter multipled by three, which was ±0.015 cm^3.

For the first experiment, we used a one-way ANOVA to test the statistical significance of the diameter and effective density values and found the p-value is equal to 0.0001. Thus, the ANOVA test result supports our alternate hypothesis that the diameter cubed of the crumpled paper ball is directly proportional to the side length squared of the paper sheet.

Since the effective density remains almost constant, the hypothesis that the diameter cubed of the crumpled paper ball is directly proportional to the side length squared of the

Table 1. Raw quantitative data for the effective density experiment, including the average diameter³ and effective density values.

Side length (cm) $\Delta L = \pm 0.05$ cm	Side length^2 (cm^2) $\Delta L^2 = \pm 0.1\ cm^2$	Avg. Diameter (cm) $\Delta D = \pm 0.005$ cm	Avg. Diameter^3 (cm^3) $\Delta D^3 = \pm 0.015\ cm^3$	Avg. Effective density (g/cm^3)
5	25	1.05	1.16	0.2971
8	64	1.42	2.84	0.3097
10	100	1.73	5.21	0.2642
12	144	2.02	8.20	0.2416
15	225	2.35	12.98	0.2385
17	289	2.80	21.95	0.1811
20	400	3.20	32.77	0.1679

paper sheet is supported.

In order to investigate the relationship between the number of times a sheet of paper was crumpled and the crumpled ball's compressive strength in the second experiment, we placed the crumpled A4 paper on the force platform and compressed it until it became crushed, and the reading on the datalogger was equal to the compressive load value.

The data was processed by first calculating the area of the A4 paper, which is 210 mm × 297 mm = 62370 mm^2. This measurement was kept constant because the crumples formed irregular shapes from which a diameter could not be determined, and thus the cross-sectional area could not be calculated. The compressive strength was found by dividing each compressive load by the area, which for the paper crumpled 2 times was:

$$Compressive\ strength = \frac{compressive\ load}{surface\ area} = \frac{36.77\ N}{62370\ m^2} = 0.00058955\ MPa = 589.55\ N/m^2 \quad (8)$$

The same process was repeated for the different crumple numbers and an additional two replicates to give three total trials (**Table 2**). The margin of error for compressive load shown in **Table 2** was derived from the systematic error from the force platform, which was half of the measuring unit. As the measuring unit was 0.01 N, the margin of error was ±0.005 N.

For the second experiment, we also used a one-way ANOVA test to test the statistical significance of the compressive load values and found the p-value is equal to 2.33E-10, significantly below the significance threshold. Thus, the ANOVA test result provided evidence disproving the null hypothesis that there is no different in a paper sheet's compressive strength when the number of times the paper is crumpled is varied. It supported the alternate hypothesis that the more times a paper sheet is crumpled, the greater its compressive strength.

Since the greater the number of times the paper was crumpled indicated a greater compressive load and compressive strength, the hypothesis that there is a directly proportional relationship was supported.

Table 2. Raw quantitative data for the compressive strength experiment, including the average compressive load and compressive strength values.

No. of times crumpled	Average Compressive load (N) ΔF = ±0.005 N	Average Compressive strength (Pa) ΔCS = ±0.005 Pa
2	36.77	589.55
3	50.84	815.19
4	99.36	1593.07
5	126.67	2030.94
6	142.52	2285.07
7	196.12	3144.46
8	266.62	4274.76
9	290.73	4661.32

DISCUSSION

Our results from the first experiment demonstrate that the effective density of the crumpled paper balls is three times less than the measured density of the paper sheet. This supports the idea that crumpled paper is mostly made up of air (2). While the diameter cubed of the paper ball varies with the side length squared of the paper sheet (**Figure 2**), the effective density decreases somewhat as the side length increases. A reason for this could be that the larger the piece of paper, the harder it is to crumple and ensure that the maximum force is being applied by hand. This would result in a larger than proportional crumpled ball and a larger volume and therefore a lower effective density.

Rearranging equation (2) from the Materials and methods section and using the slope of the trendline (which is the average thickness of the paper), the thickness can be calculated by:

$$t = \frac{\rho_e \pi d^3}{6\rho_p l^2} = \frac{\rho_e}{\rho_p} \times \frac{\Delta d^3}{\Delta l^2} \times \frac{\pi}{6} = \frac{1}{3} \times 0.0849 \times \frac{\pi}{6} = 0.015 \pm 0.030\ cm \quad (9)$$

This is quite close to the standard thickness of A4 paper, 0.01 cm, thus supporting the hypothesis as it means that equations (1) and (2) hold true, demonstrating the relationship between side length squared and diameter cubed. One reason the calculated value for thickness is a little larger than the standard thickness could be due to the fact that the 0.3097 g/cm^3 value for the average effective density when the side length is 8 cm is an outlier. Since the diameter cubed is inversely proportional to the effective density, the value for the diameter cubed at this side length may have been smaller in actuality, making the trendline slope larger due to an artificially high value. However, a Thompson-Tau test revealed that there are no outliers in this experiment, though the 289 cm^2 and 400 cm^2 values of 0.1811 g/cm^3 and 0.1679 g/cm^3 respectively are close to being outliers. If this experiment were repeated, we could measure whether the

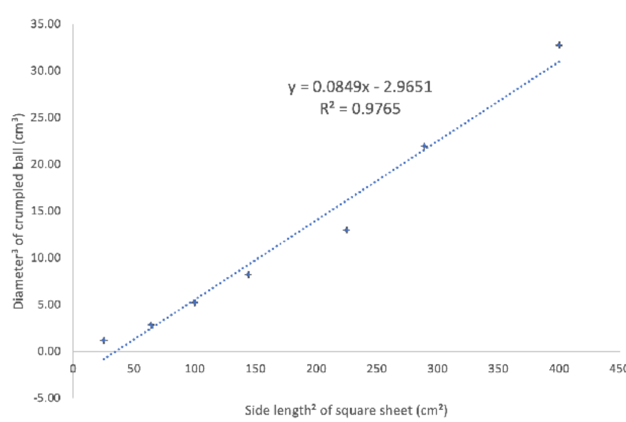

Figure 2. The graph of diameter^3 against side length^2. Statistical analysis performed by one-way ANOVA, p = 0.0001. As the errors were negligible, the error bars were not plotted.

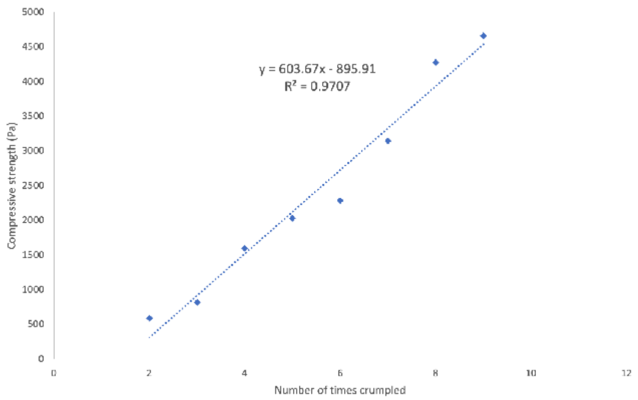

Figure 3. The graph of compressive strength against the number of times crumpled. Statistical analysis performed by one-way ANOVA, p = 2.32973E-10. As the errors were negligible, the error bars were not plotted.

Vernier calliper is applying any unwanted force to the paper ball during diameter measurements. Moreover, more side lengths could be tested to ensure that the relationship holds for large sized crumpled paper balls. Another reason for this outlier could be that the crumpling process was conducted entirely by hand, meaning that it was very hard to keep the compacting force constant for all the paper balls. For example, it may have been much easier to crumple a smaller piece of paper than a large one, as a larger one would trap more air in its crumpled form. This would mean the crumpled paper ball made from the 5 cm paper sheet was artificially denser than the ball made from the 20 cm paper sheet due to disparities in force application. To integrate our investigations of effective density and compressive strength, we could compare compressive strength for paper sheets of different side lengths to determine whether a higher or lower density of a paper ball leads to a higher compressive strength value.

In the second experiment, there is a relatively strong linear relationship between the number of times the paper was crumpled and its compressive strength (**Figure 3**). By the time the paper was folded eight times, it was almost completely crumpled into a uniform ball, so the increase in compressive strength between the paper folded eight times and the paper folded nine times was not as significant. Nonetheless, the differences in compressive strength between the paper folded four, five and six times are of similar magnitude. This positive correlation also supports the hypothesis that the more crumples there are in a piece of paper, the greater its compressive strength. Our results have interesting implications, as this means that there is a difference in structural strength between a partially crumpled piece of paper and a completely crumpled paper ball. The more crumples the paper has, the harder it is to compress due to the fact that it contains a large amount of air (2).

As mentioned in the Results section, the cross-sectional area of the partially crumpled paper balls could not be calculated because the crumples formed irregular shapes. The surface area measurement for the uncrumpled paper sheet was used instead. This is a significant limitation, as we assumed in this experiment that the area remained constant when it may not necessarily be the case. Unless a new way of crumpling paper into perfect spheres is discovered, there is no easy solution to minimise the error in the area calculations.

Since a mechanical compressor was not used in this experiment, the air remained trapped inside the paper structure and prevented the paper from becoming deformed easily. If we had used a mechanical compressor, we could expect the force applied to be constant regardless of crumple number. Another reason for fully crumpled paper's unusual compressive strength is the stacks or folds in the paper created from the crumpling process providing strength to the structure in all angles. As these multiple folded layers come together, they act as structural pillars. Since they are aligned in many different, random directions and are isotropically arranged (7), they strengthen the crumpled ball in every direction. Thus, no matter which angle one presses down on the crumpled ball, he or she is pressing down against these columns, which resist being crushed in all directions. In future experiments, we could study how the compressive strength may depend on the shape of the paper balls since different shapes, especially when the paper is only partially crumpled, trap different amounts of air, which could affect how resistant it is to compression.

In automobiles, crumple zones are areas, usually in the front or back, that are designed to deform in a collision to ensure passenger safety (8). The aim of crumple zones is to absorb as much kinetic energy as possible by reducing the initial force of the crash and redistributing the force before it reaches the vehicle's occupants. Newton's second law, $F = ma$, shows that the greater the acceleration, the greater the force. Thus, as crumple zones absorb energy to decrease acceleration by adding time to the crash, the impact experienced by the occupant will decrease. Many may believe that the stronger the materials used to make cars, the safer the vehicle. However, this isn't true as the force from a collision would immediately be felt by the occupant. Less stiff plastics and polymer fibres are often used to manufacture crumple zones as these result in the maximum crushing, ensuring the occupant is protected in the car (9). This again shows that the lower the flexural rigidity of a material, the more likely it is to become crumpled, and in this case, ensure automobile safety. Thus, a potential future area of study would be to examine whether the crumpling properties of plastics and polymer fibres are similar to those of paper and if there could be applications of how they crumple on how to best design crumple zones to maximise safety in the case of a crash.

MATERIALS AND METHODS

In the first experiment, identical sheets of A4 paper were cut into square pieces with different side lengths (10).

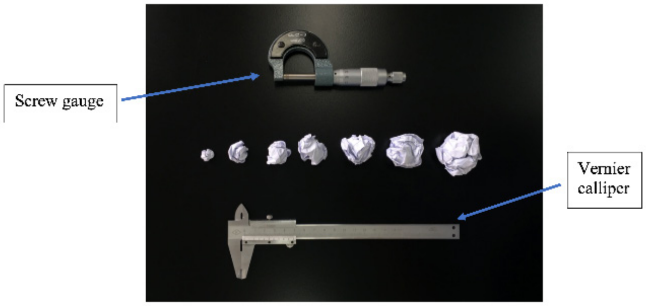

Figure 4. The different sized crumpled paper balls and apparatus set-up. The Vernier caliper and screw gauge were used to measure the diameter of the crumpled paper balls.

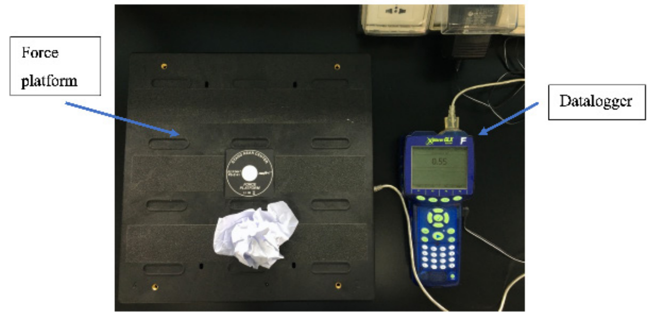

Figure 5. The paper ball on the PASCO PASPORT Force Platform and apparatus set-up, including the Xplorer GLX datalogger which was used to record the data from the force platform.

The sheets were then crumpled by hand to form the shape of a ball. These were crumpled once completely, with the maximum amount of force that could be applied by hand. We used a Vernier calliper and a screw gauge to measure the diameter of the paper balls (**Figure 4**). The variables for the first experiment are: (independent) the side length squared of the paper sheets, (dependent) the diameter cubed of the crumpled paper balls, (controlled) the compression force applied by hand and the material of the paper.

We began by equating the mass of a paper sheet to the mass of the crumpled paper balls:

$$\rho_p l^2 t = \rho_e \frac{4\pi r^3}{3} \text{ or } \rho_p l^2 t = \rho_e \frac{\pi d^3}{6} \tag{1}$$

where ρ_p is the measured density of the paper sheet, l is the side length of the square sheet of paper, t is the thickness of the paper, ρ_e is the effective density of the paper ball and d is the diameter of the paper ball. Rearranging, we have:

$$d^3 = \frac{\rho_p 6 t l^2}{\rho_e \pi} \tag{2}$$

Since $d^3 \propto l^2$, we hypothesized that the diameter cubed of the crumpled paper ball is directly proportional to the side length squared of the paper sheet. On the other hand, the null hypothesis is that there is no difference in diameter cubed of the paper ball when the side length squared of the paper sheet is varied.

In the second experiment, one crumple was defined as one squeeze of the paper with two hands. Since it was hard to get definite crumpling with just one crumple (the paper tended to fold itself rather than crumple), the smallest value for the number of times crumpled was two, and the greatest value was nine. By nine crumples, the paper was completely crumpled into a ball. We first crumpled the A4 paper two times then placed it on the PASCO PASPORT Force Platform, which was connected to the Xplorer GLX datalogger (**Figure 5**). The crumpled paper structure was compressed until it deformed (i.e., it became crushed). The data output was compressive load. This process was repeated for the number of paper crumples from three to nine crumples two times to give three trials. The variables for the second experiment are: (independent) the number of times the A4 paper is crumpled by hand, (dependent) the compressive strength of the crumpled paper balls, (controlled) the type and size of the A4 paper and the force applied per crumple.

The one-way ANOVA tests were conducted using the StatPlus computer application for both experiments, and the significant threshold we used was p < 0.05. The equations used were:

$$F = \frac{MST}{MSE} \tag{3}$$

$$MST = \frac{\sum_{i=1}^{k}\left(\frac{T_i^{\,2}}{n_i}\right) - \frac{G^2}{n}}{k-1} \tag{4}$$

$$MSE = \frac{\sum_{i=1}^{k}\sum_{i=1}^{k} Y_{ij}^{\,2} - \sum_{i=1}^{k}\left(\frac{T_i^{\,2}}{n_i}\right)}{n-k} \tag{5}$$

where F is the variance ratio for the test, MST is the mean square due to treatments between groups, MSE is the mean square due to error within groups, Y_{ij} is an observation, T_i is a group total, G is the grand total of all observations, n_i is the number in group i and n is the total number of observations.

Received: June 16, 2020
Accepted: November 16, 2020
Published: December 2, 2020

REFERENCES

1. Hanaor, D., *et al.* “Mechanical properties in crumple-formed paper derived materials subjected to compression.” *Heliyon*, vol. 3, no. 6, June 2017, doi:10.1016/j/heliyon.2017.e00329.
2. Croll, Andrew B., *et al.* “The compressive strength of crumpled matter.” *Nature Communications*, vol. 10, 3 Apr. 2019, doi:10.1038/s41467-019-09546-7.
3. Roberts, Siobhan. “How scientists are using crumpled paper to better understand DNA.” *Independent*, 12

Dec. 2018, www.independent.co.uk/news/science/dna-physics-crumpling-paper-science-solar-sail-satellite-how-what-history-a8666526.
4. Keim, Brandon. "The cutting-edge physics of a crumpled paper ball." *Wired*, 22 Aug. 2011, www.wired.com/2011/08/crumpled-paper-physics.
5. Bansal, Tarush, *et al*. "Effective density of crumpled paper balls." *Journal of Basic and Applied Engineering Research*, vol. 5, no. 2, 2018, pp. 123-125.
6. Cambou, Anne D. *On the crumpling of thin sheets.* Diss. UMass Amherst, 2014. Web. 13 June 2020.
7. Cambou, Anne D., and Narayanan Menon. "Three-dimensional structure of a sheet crumpled into a ball." *Proceedings of the National Academy of Sciences of the United States of America*, vol. 108, no. 36, 6 Sept. 2011, pp. 14741-14745, doi:10.1073/pnas.1019192108.
8. Grabianowski, Ed. "How Crumple Zones Work." HowStuffWorks, 11 Aug. 2008, auto.howstuffworks.com/car-driving-safety/safety-regulatory-devices/crumple-zone.htm.
9. *Crumple Zone Physics - How Plastics in Cars Can Save Lives*. Automotive Plastics, 29 Aug. 2019, www.automotiveplastics.com/blog/physics-in-the-crumple-zone-demonstrate-how-less-stiff-materials-like-plastic-can-help-prevent-injury-and-save-lives/.
10. Tallinen, T., *et al*. "The effect of plasticity in crumpling of thin sheets." *Nature Materials*, vol. 8, Jan. 2009, pp. 25–29, doi:10.1038/nmat2343.