

Determining the Hubble constant through the analysis of Abell clusters

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SUMMARY

The Hubble constant (H_0) quantifies the present-day expansion rate of the Universe and is a fundamental parameter for determining its age; however, modern cosmology faces significant tension, as increasingly precise measurements continue to yield mutually inconsistent values. The two main methods of measuring H_0 , namely Cepheid variable stars and the Cosmic Microwave Background (CMB), have produced inconsistent results. Here, we present an alternative method to measure the expansion rate of the universe using galaxy clusters. We hypothesize that the relationship between cluster's redshift and galaxy magnitude can yield a linear relation and produce a measurable value for H_0 , assuming the fifth-brightest galaxy in each galaxy cluster has a consistent luminosity. Utilizing data from the Sloan Digital Sky Survey (SDSS) and the Abell Cluster Catalogue, redshifts were derived from SDSS optical spectra, and distances to the clusters were estimated under the assumption that member galaxies share similar brightness. The analysis yielded a H_0 value of 73.9 ± 11.4 km/s/Mpc, and the measurement, within uncertainty, remains consistent with the two leading methods of measuring H_0 . This finding suggests that expanding the dataset to include more galaxy clusters is required to improve the accuracy of H_0 measurements.

INTRODUCTION

Since the Big Bang roughly 13.8 billion years ago, the universe continues to expand, a fundamental aspect of cosmology that provides crucial insights into its large-scale structure and fate (1). In 1929, Edwin Hubble discovered that galaxies recede from one another, revealing a linear relationship between their distance and redshift (2). This observation formed the foundation for the Hubble constant (H_0), which characterizes the rate of cosmic expansion. Defined as the proportionality constant in Hubble's law, H_0 relates a galaxy's recession velocity (v) to its distance from Earth (d), where recession velocity is the speed at which a galaxy moves away from the observer due to the expansion of the Universe: $v = H_0 d$ (3). Following Hubble's initial observations, astronomers have sought to refine measurements of H_0 using a variety of methods, including but not limited to Cepheid variable stars, Type Ia supernovae, and the Tully-Fisher relation. Each technique offers distinct advantages and limitations, contributing to the ongoing debate regarding the precise value of H_0 .

Among these, one of the most widely used methods is

measuring galaxy distances using Cepheid variable stars, whose luminosity varies in a predictable manner over time (4). By comparing the apparent brightness of these stars to their known intrinsic luminosity, astronomers determine their distances and combine this with redshift data to calculate the value of H_0 . Redshift (z) refers to the stretching of a galaxy's emitted light longer (redder) wavelengths due to the expansion of the Universe and provides a measurement of its recession velocity. Another prominent approach derives H_0 from observations of the Cosmic Microwave Background (CMB), which constrains the expansion rate of the early universe (5). In recent years, the Planck satellite has provided high-precision measurements of the CMB, yielding a precise estimate of H_0 based on early-universe physics (5).

Despite advances in both technology and methodology, significant discrepancies remain between different measurements of H_0 . For example, the Cepheid variable stars method yields a value of approximately 72 ± 1 km/s/Mpc, while measurements based on the CMB suggest a lower value, around 67 km/s/Mpc (5,6). These conflicting results raise the possibility that either method suffers from systematic uncertainties. For instance, an incorrect assumption about the luminosity prediction of Cepheid variable stars would highlight limitations in our astrophysical techniques. Alternatively, the discrepancy may point to new physics, suggesting that our current understanding of the expansion of our universe requires revision or extension (4).

Another possible method for measuring H_0 involves analyzing the redshift of galaxy clusters (7,8). As the largest gravitationally bound structures in the universe, galaxy clusters contain hundreds to thousands of galaxies embedded in massive halos of dark matter and hot gas, spanning millions of light-years (9). Their enormous mass generates strong gravitational fields that influence both the surrounding space and the propagation of light (10). Given their large masses and sizes, galaxy clusters serve as an excellent candidate for studying cosmic expansion (11). The redshift of a cluster provides a direct measurement of its receding velocity relative to Earth, while its distance can be estimated through the apparent magnitudes of its member galaxies. This relationship between distance and velocity is fundamental to determining H_0 , which quantifies the rate of the universe's expansion.

Galaxy clusters offer several advantages for measuring H_0 compared to traditional methods, such as Cepheid variables or Type Ia supernovae. Unlike individual stars or supernovae, clusters are less affected by local variations in stellar population or interstellar medium properties, providing a more stable and consistent measure over large cosmological

distances (11). Furthermore, because galaxy clusters are distributed across a wide range of cosmic environments, they enable a more comprehensive sampling of the universe's expansion history (11). These characteristics make them a valuable and practical tool for estimating H_0 , particularly when used in conjunction with other observational techniques (12, 13).

Herein, we proposed using galaxy clusters as an alternative method for determining the H_0 . Our goal was to assess whether this measurement is consistent with those derived from Cepheid variable stars or the CMB methods. Resolving the discrepancies between these methods is essential for improving our understanding of the universe's expansion. Specifically, we investigated whether redshifts of galaxy clusters be used to determine the H_0 value. We hypothesize that if the fifth brightest galaxy in each cluster exhibits a consistent intrinsic luminosity, then the relationship between the cluster's redshift and the magnitude of that galaxy will be linear, allowing for an independent determination of H_0 . Overall, our results are consistent with both Cepheid- and CMB-based method of H_0 within current uncertainties; however, improved statistics from larger cluster samples will be required to meaningfully discriminate between these approaches. Our results provide an independent perspective on the H_0 debate and contribute to ongoing efforts to reconcile the differing values obtained from the other two measurements.

RESULTS

We investigated whether galaxy clusters can be used as reliable indicators for estimating the Hubble constant. Given galaxy clusters are massive, distant systems with measurable

redshifts and magnitudes, they offer an alternative method from traditional distance indicators such as Cepheid variables and Type Ia supernovae (14). To test this, we analyzed a sample of nearby (low redshift) Abell galaxy clusters using publicly available Sloan Digital Sky Survey (SDSS) data. For each cluster, we estimated the distance from the apparent magnitude of the fifth-brightest galaxy, assuming that these galaxies share approximately standard intrinsic luminosities (**Figure 1**). By plotting the redshift-derived recession velocities against the estimated distance, we performed a linear fit using orthogonal distance regression (ODR). The resulting slope of the best-fit line, 73.9 ± 11.4 km/s/Mpc, represents our calculated value for H_0 .

We first calculated the average redshift of the 12 Abell clusters (**Table 1**). The relationship between the distance of each Abell galaxy cluster was calculated from the magnitude of the fifth brightest galaxy in each cluster and its redshift (**Figure 2**). The linear fit between distance and velocity yields a slope of 73.9 ± 11.4 km/s/Mpc, corresponding to the Hubble constant, H_0 .

To quantitatively assess whether the derived H_0 values differed significantly across photometric bands or galaxy rank selections, we performed a one-way Analysis of Variance (ANOVA) on the fitted slopes obtained from each configuration. The ANOVA test comparing the photometric bands (g, r, and i) yielded no statistically significant difference among the slopes (f -value of 1.44 and p -value = 0.2518). Similarly, no significant difference was found when comparing slopes derived from the third, fourth, and fifth brightest galaxies (f -value of 2.41 and p -value = 0.1054). These results support the consistency of the derived H_0 values across band choice and galaxy selection.

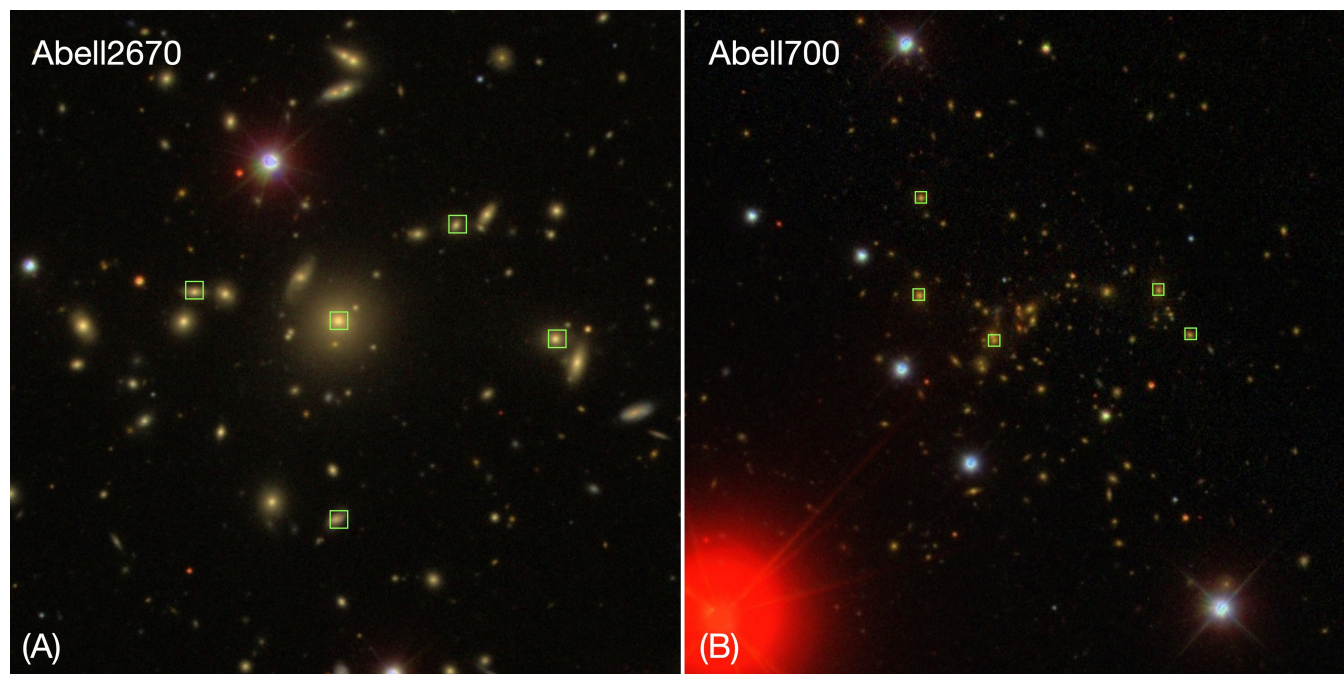


Figure 1. Example images of Abell Cluster from Sloan Digital Sky Survey (SDSS). (A) SDSS optical image of Abell2670 ($z = 0.080$). (B) SDSS optical image of Abell700 ($z = 0.280$). The image of Abell700 clearly indicates that it is visible farther away than Abell2670 because the size of each galaxy is smaller in Abell700. Green labels mark the five cluster-member galaxies used to derive each cluster's systemic redshift, selected based on their similar colors and apparent sizes.

Name	RA (deg)	DEC (deg)	Redshift Mean	Redshift Uncertainty
Abell 0085	10.4075	-9.3425	0.052	0.005
Abell 0779	139.9550	33.7603	0.021	0.003
Abell 1132	164.5800	56.7822	0.136	0.002
Abell 1377	176.7413	55.7389	0.052	0.003
Abell 1650	194.6926	-1.7530	0.084	0.003
Abell 2034	227.5547	33.5279	0.112	0.006
Abell 2199	247.1540	39.5243	0.029	0.003
Abell 2670	358.5423	-10.4050	0.080	0.008
Abell 2197	247.0481	40.8988	0.032	0.001
Abell 2255	258.1212	64.0965	0.081	0.003
Abell 1218	169.7037	51.7349	0.081	0.001
Abell 1387	177.1953	51.6164	0.132	0.002
Abell 700	131.5005	36.9653	0.280	0.004
Abell 1995	223.2053	58.0678	0.323	0.005

Table 1: List of Abell Clusters used in the study. Properties of low-redshift Abell Clusters, including their celestial coordinates Right Ascension (RA) and Declination (Dec), redshift estimates, and associated uncertainties, obtained from SDSS (18).

Using the third and fourth brightest galaxies instead of the fifth produces very similar distance–velocity trends (**Figure 3**). The slopes remain consistent within the uncertainties, suggesting that the result is not strongly affected by which of these bright cluster galaxies is chosen. The distance–velocity relationship is maintained across the g, r, and i photometric bands (**Figure 4**). Although the fitted slopes vary slightly between bands, they agree within uncertainties, indicating that the result is not sensitive to band choice.

DISCUSSION

We calculated a H_0 measurement of 73.9 ± 11.4 km/s/Mpc, which is consistent within uncertainties with values obtained from other methods. For example, the supernova technique

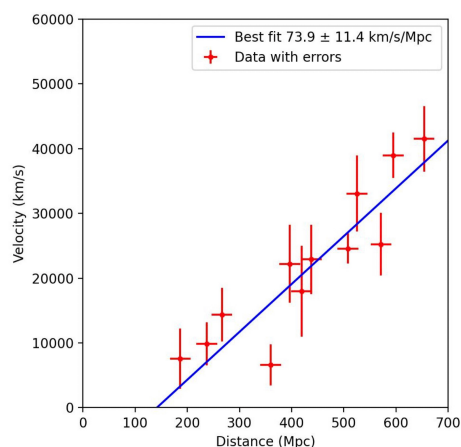


Figure 2. Linear relationship between the distance and velocity of galaxies. The distance (Mpc) versus velocity (km/s) for 12 Abell galaxy clusters (red points), excluding two outliers. The blue line represents a linear fit of the data, with the slope corresponding to the H_0 value. Both the g-band magnitude and the redshift are obtained from SDSS.

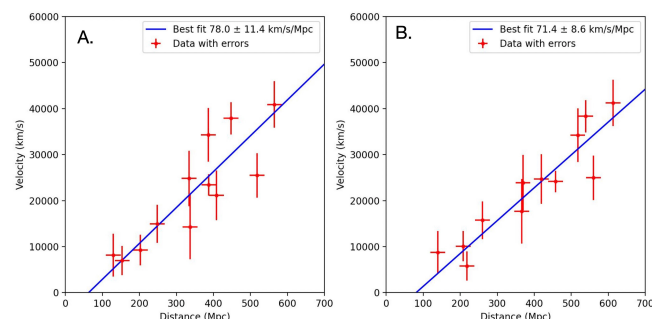


Figure 3. Linear relationship between distance and velocity using different reference galaxies. The figures show the distance–velocity relation for 12 Abell cluster, using A) the third and B) the fourth brightest galaxies in each cluster. In both panels, the blue lines represent linear fits to the data, with slopes corresponding to the inferred H_0 values. The consistency of the fitted slopes across both selections indicates that the result is not strongly dependent on the choice of reference galaxy.

gives H_0 approximately 73.04 ± 1.04 km/s/Mpc, while CMB measurements from the Planck satellite suggest 67 km/s/Mpc (3,5). However, the uncertainty in our measurement remains too large to distinguish between these approaches or to favor one over the other. Future improvements are needed to reduce this uncertainty, particularly by refining the assumptions underlying our method.

Despite these limitations, this approach offers several advantages. It provides an independent determination of H_0 that does not rely on the traditional distance ladder calibration from Cepheids to Type Ia supernovae. Instead, it leverages observable properties of galaxy clusters, which are abundant and can be identified out to high redshift in large survey datasets. This makes the method potentially scalable with future surveys, offering a complementary avenue to test the robustness of existing measurements. As larger and more homogeneous cluster samples become available, this technique may provide increasingly competitive constraints and contribute to resolving the current tension in H_0 measurements.

To understand the origin of these uncertainties, it is

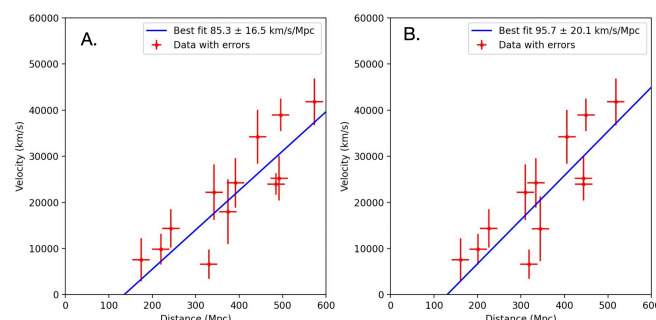


Figure 4: Linear relationship between distance and velocity using different photometric bands. The figures show the distance–velocity relation for 12 Abell cluster, using brightness measurement from the SDSS A) r and B) i band for each galaxy. In both panels, the blue lines represent linear fits to the data, with slopes corresponding to the inferred H_0 values. The consistency of the fitted slopes across both selections indicates that the result is not strongly dependent on the choice of photometric bands.

important to examine the assumptions underlying our method. Several assumptions underlie the calculations in this study. The primary assumption is that the fifth brightest galaxy in each cluster has a luminosity comparable to that of the Milky Way. While this may not hold universally, it provides a practical baseline for distance estimation. To test the robustness of this approach, alternative selections, such as the third or fourth brightest galaxy, were also considered. The results remained consistent, indicating that the choice of representative galaxy does not significantly affect the derived value of H_0 .

Another assumption involves the use of g-band magnitudes, chosen for their comparability across clusters. To examine the effect of band selection, the r- and i-bands were also analyzed (**Figure 4**). The relationship between redshift and magnitude remained approximately linear in all bands, with only slight variations in the resulting H_0 values. These differences lie within the uncertainties, suggesting consistency across photometric bands. Nonetheless, additional data are needed to better constrain the slope of H_0 and to determine whether variations among bands reflect true physical differences or statistical fluctuations.

Some limitations of our study include the following: the number of galaxy clusters studied is small, resulting in considerable uncertainty in the final measurement of the H_0 . We expect that analyzing more galaxy clusters would reduce the uncertainty of our results. In addition, our analysis is restricted to low-redshift clusters, and it remains unclear whether this method is applicable at higher redshifts. Further studies will be required to test whether the high redshift clusters are suitable for this type of study. Another limitation is the utilization of the SDSS catalogue, which constrain the number of galaxies available in each cluster due to the survey's observational limits. Expanding the analysis to include other cluster catalogs, such as those from the South Pole Telescope, would allow for a more comprehensive test across a wider redshift range (15). Finally, comparing this approach with other independent methods for measuring H_0 , such as gravitational lensing or gravitational wave, would provide further insight into the current discrepancies among different techniques (16,17).

In summary, we estimated H_0 to be 73.9 ± 11.4 km/s/Mpc based on redshift measurement of 14 massive, low redshift Abell galaxy clusters. The results remain consistent with the two existing primary methods, supernova and CMB techniques. Further improvement could be achieved by increasing the sample size of galaxy clusters within a similar redshift range. Another possible way to improve this result is to use precise spectroscopic redshifts for individual member galaxies. While this work provides an independent constraint on H_0 , the current uncertainty remains too large to distinguish between existing measurement approaches. Continued efforts with larger and more precise dataset will be essential to better constrain the Hubble constant and improve our understanding of the universe's expansion.

MATERIALS AND METHODS

Data source

The primary data for this study were sourced from the Sloan Digital Sky Survey (SDSS), a comprehensive astronomical survey providing spectroscopic data on galaxies, stars, and

other celestial objects (18). The SDSS uses a 2.5-meter optical telescope to gather data, and the specific dataset utilized for this research is Data Release 18 (DR18). The SDSS was chosen for its extensive nature and the availability of its datasets, which are freely accessible online for research purposes.

Data collection

The Abell Cluster Catalogue lists numerous galaxy clusters and their parameters, including right ascension (R.A.) and declination (Dec) (19). The first version of this catalogue contained 58 clusters, with further versions expanding its coverage, including the second version with 2,712 clusters and the latest version with 4,073 clusters (20). In this study, 14 massive ($M > 10^{14}$ Msun) and low-redshift Abell clusters were selected because highly massive and bright clusters are easy to observe with the current generation of telescopes. The SDSS provides data across five magnitude bands (u, g, r, i, z), which are filters that measure galaxy brightness at different wavelength of light. The g band was chosen for analysis, as most galaxies in the g band tend to have similar properties. In different magnitude bands, galaxies might not have the exact brightness, which could make the H_0 estimate incorrect when assuming that all galaxies have the same brightness to estimate their distance (21). SQL was used for data retrieval to allow the efficient gathering of necessary data without manual collection. For each galaxy cluster, galaxies with available spectra in the SDSS database were selected to determine their redshifts.

To ensure reliability, the redshifts from five galaxies per cluster were gathered, and each cluster's average redshift was calculated to account for individual variations (**Figure 1**). This approach provided a more precise estimate of the cluster's redshift. Moreover, the galaxies were filtered based on their redshift, ensuring that selected galaxies were within a specified range ($|\Delta z| < 0.01$) of the cluster's redshift to verify that these selected galaxies were indeed the members of the clusters.

During the analysis, two galaxy velocities (Abell 700: 1,350.24 km/s and Abell 1995: 2,156.16 km/s) deviated from the expected linear trend and were identified as outliers. These may be outliers because they are the highest redshift objects in the samples, 0.28 and 0.32, respectively. Higher redshift objects tend to be smaller as they do not have enough time to grow into more giant galaxies (22). For this analysis, these outliers were removed before proceeding with the linear fit when estimating the value of H_0 .

Many variables were controlled to deliver valid H_0 measurements in this study. First, the same type of galaxies was chosen for each studied cluster. Specifically, red elliptical galaxies were selected, with redshifts in the same range ($|\Delta z| < 0.01$) as other cluster members. Secondly, SDSS data were queried using SQL to retrieve the magnitudes and classifications (galaxies or stars) of objects within a 5-arcminute radius of each cluster. This radius was selected to ensure sufficient data density while minimizing contamination from unrelated objects. The retrieved data included redshift, magnitude, and galaxy-star classification information essential for subsequent analyses. Lastly, the fifth-brightest galaxy in each Abell cluster was chosen to represent a typical

galaxy for measuring their cluster velocity and distance. The brightest galaxies in each cluster can vary substantially in brightness due to processes such as galaxy mergers and energy output from their central black holes. On the other hand, fainter galaxies are harder to measure accurately and are likely to be missed in observational surveys (23). The fifth brightest galaxy provide a useful balance: it is bright enough to be reliably detected in all clusters, while also showing relatively small variation in intrinsic brightness. This makes it a practical choice as an approximate standard candle for estimating distances in low-redshift galaxy clusters. The Python script used for all data processing and analysis is available to ensure transparency and reproducibility of the analysis (**Appendix**).

Data analysis

The measurement of H_0 is grounded in the relationship between redshift and velocity. A galaxy's redshift (z) is expressed as $z = v/c$, where v is the velocity of the galaxy receding from Earth (m/s) and c is the speed of light (m/s). Using this relationship, the velocity of each galaxy was calculated based on its observed redshift. The distance to each galaxy was derived using the relation between apparent magnitude (m), absolute magnitude (M), and distance (d): $m - M = 5 \log_{10}(d) - 5$. It was assumed that the fifth brightest galaxy in each cluster had the same absolute magnitude as the Milky Way. Specifically, the absolute magnitudes of the Milky Way in the SDSS filters (g, r, i, z) were obtained from a review by Bland-Hawthorn and Gerhard (24). This assumption provided a baseline for estimating distances because most member galaxies, which are not the brightest ones, typically have the same shape and size in clusters. This assumption was tested by picking the third or the fourth brightest galaxy instead and checking for consistency.

For the uncertainty estimates, the distance uncertainty was taken to be 20 Mpc for all clusters. This value represented a systematic uncertainty arising from the assumption that the fifth brightest galaxy in each cluster has a fixed absolute magnitude. The intrinsic scatter in the luminosity of the fifth-brightest cluster galaxies is approximately 0.2 magnitudes, which corresponded to an uncertainty of roughly 10% in distance (25). For a typical cluster distance of ~ 200 Mpc, this translated to an uncertainty of approximately 20 Mpc. The velocity uncertainty was estimated from the standard deviation of the galaxy members' redshifts in the cluster. These uncertainties were applied uniformly to all data points and propagated through the fitting procedure.

The Hubble constant was determined through Orthogonal Distance Regression (ODR) fitting of velocity versus distance data. ODR fitting accounts for uncertainties in both variables and was chosen over traditional least-squares fitting due to significant uncertainties in distance and velocity measurements (26). The independent variable for the fit was the distance of each cluster, while the dependent variable was the velocity obtained from the redshift. The slope of the linear fit represented the value of H_0 .

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APPENDIX

Python script used in this project:

```
import pandas as pd
import matplotlib.pyplot as plt
import numpy as np

clustername=['Abell_0085','Abell_0779','Abell_1132','Abell_1
377','Abell_1650','Abell_2034','Abell_2199','Abell_2670','Abel
l_2197','Abell_2255','Abell_1218','Abell_1387','Abell_700','Ab
ell_1995']

#Main
band = 'i'
brightest = 5

def main(band, brightest):
    clusters={}
    for i in range(len(clustername)):
        print(i, clustername[i], allredshift[i])
        df = pd.read_csv(dir_home / f'{clustername[i]}.
csv',skiprows=1)
        df_GALAXY=df[df['class']=='GALAXY'].sort_
values(by=f'modelMag_{band}')
        clusters[clustername[i]]={'member':df_GALAXY[np.
abs(df_GALAXY['redshift']-allredshift[i])<0.01].iloc[brightest-
1], 'std':np.std(df_GALAXY[np.abs(df_GALAXY['redshift']-
allredshift[i])<0.01]['redshift'])}

    alldistance,allvelocity=[],[]
    alldistance_error,allvelocity_error=[],[]
    for name in clusters.keys():
        alldistance.append(calculate_distance(clusters[name]
['member'][f'modelMag_{band}'],band))
        allvelocity.append(calculate_velocity(clusters[name]
['member']['redshift']))
        allvelocity_error.append(calculate_
velocity(clusters[name]['std']*5))
        alldistance_error.append(20)

    # Prepare the data for ODR
    x_data=np.array(alldistance)
    y_data=np.array(allvelocity)
    y_error=np.array(allvelocity_error)

    x_error=np.array(alldistance_error)
    data = RealData(x_data[x_data<1000], y_data[x_
data<1000],sy=y_error[x_data<1000],sx=x_error[x_
data<1000])

    # Define the model
    linear_model = Model(linear_model_odr)

    # Set up ODR with the model and data
    odr = ODR(data, linear_model, beta0=[50, 0]) # Initial
guesses for slope (m) and intercept (b)

    # Run the ODR fit
    output = odr.run()
```

```
# Get the best-fit parameters and their uncertainties
m_best, b_best = output.beta
m_uncertainty, b_uncertainty = output.sd_beta

plt.figure(figsize=(5,5))
# Plot the data with error bars and the best-fit line
plt.errorbar(x_data, y_data, yerr=y_error, xerr=x_error,
fmt='.', label="Data with errors", color='red')
plt.plot(xs, m_best * xs+ b_best, label=f"Best fit {m_best:.1f}
± {m_uncertainty:.1f} km/s/Mpc ",color='blue')
plt.xlabel("Distance (Mpc)")
plt.ylabel("Velocity (km/s)")
plt.legend()
plt.xlim(0,700)
plt.ylim(0,60000)
plt.title(f'{band} band, {brightest}th brightest galaxy')
plt.tight_layout()
plt.savefig(f'{band}band_{brightest}th_brightest.
jpeg',dpi=200)
plt.close()
```