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Modeling Energy Produced by Solar Panels

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Summary

As solar panels become increasingly popular, consumers want to know how to harvest more of the sun's energy and how to increase the efficiency of solar panels. Determining the annual optimum tilt angle of a solar panel is a simple yet effective way to increase the energy generated. The hypothesis was that if the tilt angle of the solar panel was set to the latitude of the location, then the solar panel would generate the most energy annually. A mathematical model was built to simulate the amount of solar energy generated in a day and in a year and was used as comparison to field data. Ten solar panels of varying angles were set outside and the voltage across a resistor was logged every five minutes to measure the energy generated by the solar panels. The model simulation supported the hypothesis, as the solar panel tilt angle of 40° generated the most energy and the latitude of the location of the experiment was around 40°. The field data supports the hypothesis at the experimental latitude and the results of the simulation confirm the field data, and shows that the solar panel tilt angle should be set within ±10° of the latitude to produce the most energy.

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Introduction

Fossil fuels are one of the largest contributors to climate change because when burned, they release carbon dioxide and other gases into the atmosphere. Carbon dioxide currently makes up 65% of greenhouse gas emissions (1). In addition, fossil fuels are a finite source of energy and our current fossil fuel sources may be depleted by 2088 (2). As technology advances, everyday consumers begin to recognize the benefits of renewable energy and begin to implement them in their homes. One of the most popular forms of renewable energy is solar energy (3).

The concept of solar energy began as simple additional windows on the sunnier side of one's house. Scientists started to experiment more with solar energy and in 1954, Daryl Chapin, Calvin Fuller, and Gerald Pearson invented the first silicon photovoltaic cell (4). The photovoltaic cell converted solar radiation directly into electricity using a mechanism where photons knock the electrons free from a silicon semiconductor. The electrons travel through a series of conductors and generate a flow of direct current (4). However, despite the revolutionary invention of the photovoltaic cell, the low efficiency and high cost of solar panels discouraged many customers from applying this eco-friendly method in their own homes.

There are two ways to lower the cost and increase the efficiency of solar panels. First, material scientists have experimented with different methods such as using inexpensive semiconductor material (perovskites (5) and monocrystalline solar cells (6)) or developing intermediate band photovoltaics (7), to lower the cost and increase the efficiency (8). The second, and more feasible way to increase the efficiency of solar panels is to ensure that the solar panel is set to the optimum tilt angle where the solar panel is most often perpendicular to the sun's rays and will receive the highest intensity of sun rays. This is the most practical and efficient way to optimize the energy generated by the solar panels. There are multiple ways to optimize the tilt angle of the solar panels. An expensive method is to install a solar tracker on the roofs of houses that follows the sun as it travels

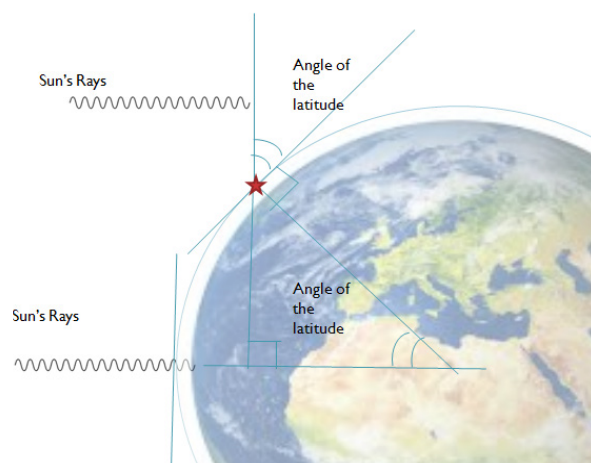

Figure 1. Diagram of Sun's Rays on Earth. If the sun's rays have an average position of being directly perpendicular to the equator, then the solar panels in a specific location must be set to the angle of the latitude for the sun's rays to be most often perpendicular to the average position of the sun's rays.

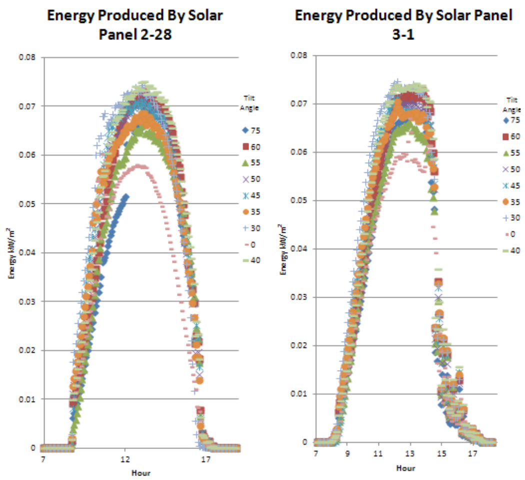

Figure 2. Field Data Produced on February 28 and March 1, 2016. Energy produced by solar panels at each tilt angle tested. The angle producing the most energy is 40° (the angle of the latitude of the location tested). Additionally, 0° is producing the least amount of energy, as expected.

through the sky, that way the solar radiation will always be perpendicular to the solar cell. The more practical way to optimize the tilt angle is to set the solar panel to a constant angle that will be most often perpendicular to the sun's rays. This work aims to develop a mathematical model to determine the optimum tilt angle and use field data to evaluate the theoretical calculations of solar energy harvested by solar panels.

It was hypothesized that if the tilt angle of the solar panel is set to the latitude of the location, then the solar panel will generate the most energy annually because that is when the solar panel will be most often perpendicular to the sun's rays. If the sun's rays have an average position of being directly perpendicular to the equator, then the solar panels in a specific location must be set to the angle of the latitude for the sun's rays to be most often perpendicular to the average position of the sun's rays. (**Figure 1**)

To test this hypothesis, a mathematical model calculating the amount of solar energy that can be generated at any tilt angle was created with the Java programming language using the BlueJ development environment (9). In addition, field measurements were collected with 10 solar panels at varying angles. These 10 solar panels were placed outside to investigate if the solar panels with the tilt angle of the latitude truly produced the optimum energy.

Results

To test the accuracy of the mathematical model, field measurements with 10 solar cells from Sundance Solar were performed. Each of the 10 solar panels were set to angles of 0°, 15°, 30°, 35°, 40°, 45°, 50°, 55°, 60°, 75° and positioned to face south (azimuth angles of each solar is 180 °) and a data logger recorded the energy that the solar panels produced every 5 minutes. The solar panels were set outside from sunrise to sunset from February 28 to March 23, 2016. The solar panel set to 40° produced the most energy of all the angles tested during the diurnal cycle between February 28 and March 1st (**Figure 2**). The hourly energy measured from the field data follows a diurnal cycle similar to that predicted by the mathematical model, increasing as solar noon is approached and decreasing as the sun sets in the sky (**Figure 3**). The inputs of the model include latitude, day number, azimuth angle (180°), solar cell efficiency, solar cell area, and solar panel tilt angle. The model simulated the hourly solar energy from a solar panel tilted to 40° at 39.27° latitude on February 28th.

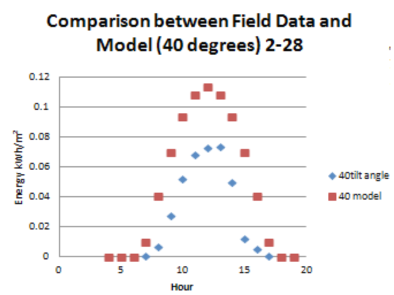

Figure 3. Comparison between Field Data Solar Panels and Results from Model at 40°. Energy generated throughout the day on February 28th for the field measurements and the mathematical model. Both follow the diurnal cycle of the sun, producing less energy in the morning, reaching its peak at noon and decreasing in the evening.

The experiments were conducted near the spring equinox (March 21st), which means that the sun is directly over the equator. When the sun's rays are perpendicular to the Equator, they are in the average annual position, as shown in **Figure 1**. Since the field experiments and model simulation were performed near the sun's annual position, the data can represent sun's average annual performance. The data shows that 40° is the optimum annual tilt angle which supports the hypothesis stating that the optimum tilt angle is equal the latitude of the location, 39.27°. Solar panels set at angles near the latitude angle 40° produced the greatest amount of daily energy in kWh, as seen in **Figure 4**.

According to the mathematical model's calculation of annual energy as a function of tilt angle, a 40° tilt angle will produce the highest amount of solar energy. However, the angles within ± 10° produce close to the amount of energy 40° produced, 295 kWh/m² (**Figure 5**). Additionally, the tilt angle that generates the most energy annually is also at the angle of the latitude (**Figure 6**). The maximum value produced at each latitude is highlighted with a black star. For example, at a latitude of

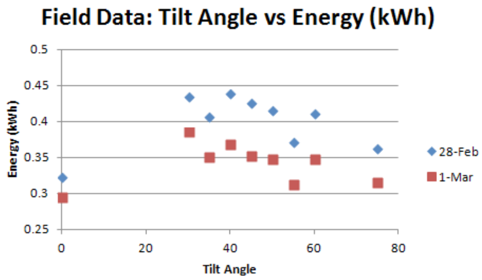

Figure 4. Comparing Solar Panel Tilt Angle and Energy from Field Data Calculations. The tilt angle is plotted against the amount of energy in kWh it produced to show that angles near the latitude angle, 40°, produced the most amount of energy.

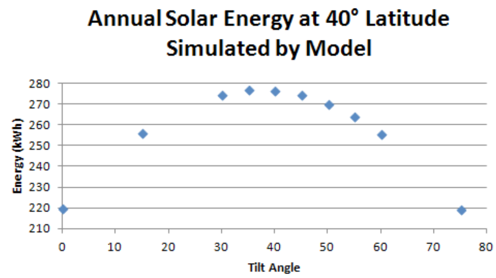

Figure 5. Annual Solar Energy Simulated by the Mathematical Model at 5 Different Latitudes. This plot graphs tilt angle versus annual energy simulated from the mathematical model. According to the mathematical model's calculation of annual energy, 40° tilt will produce the highest amount of solar energy. Angles within +/- 10° produce close to the amount of energy 40° produced, 295 kWh/m².

30°, the solar panel set to 30° produces the most energy, 292.0816 kWh. 40° is the annual optimum tilt angle for the location where the experiment was conducted, 39.27° latitude.

As seen in **Figure 3**, the model overestimated the amount of energy recorded in the field data by about 40%. This difference is likely due to the simplified version of irradiance calculation in the mathematical model (10). The formula used to calculate solar irradiance is a function of air mass, which is affected by solar zenith angle only. In reality, when solar beams pass through the atmosphere, they can be attenuated due to atmospheric scattering and absorption. Solar irradiance at the Earth's surface varies with atmospheric conditions because thin, wispy clouds and certain atmospheric qualities such as the water vapor, atmospheric pressure, ozone, and albedo may scatter or absorb sunlight and reduce the amount of sunlight transferred to the Earth's surface (11). A complicated atmospheric radiative transfer model is required to fully take into account all the effects from the atmosphere. The empirical formula used in the calculations here likely underestimated the scattering and absorption of sunlight in the atmosphere, which is why the model overestimated the energy produced by the solar panel.

Discussion

The first step in making renewable energy popular is to make it more efficient and affordable. According to the results from the field data, the annual optimum tilt angle of solar panels is equal to the latitude of the location, because the tilt angle that produced the most energy was 40° and the latitude of location of the experiments was 39.27°. This result is confirmed by the mathematical model which also shows that 40° should produce the most annual energy. However, both model and data

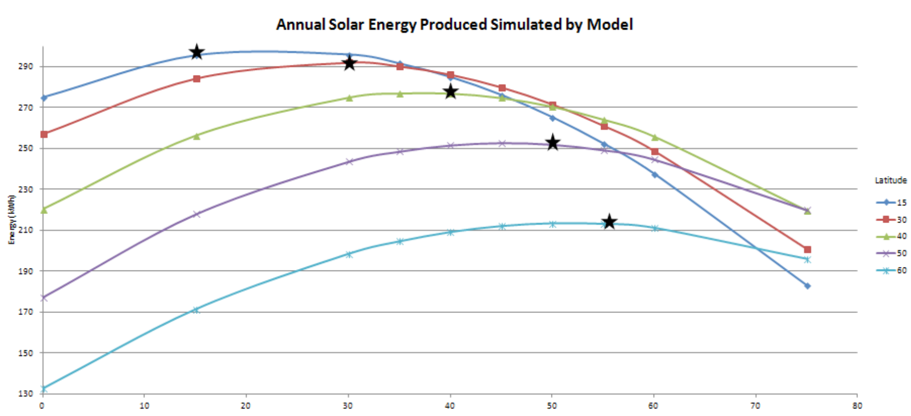

Figure 6. Annual Solar Energy Produced at Different Latitude Simulated by the Mathematical Model. Annual solar energy generated at different latitudes to demonstrate that the optimum tilt angle is the angle of the latitude. The maximum value produced at each latitude is highlighted with a red dot. For example, at a latitude of 30°, the solar panel set to 30° produces the most energy, 292.0816 kWh.

suggest that there isn't a large difference between angles that are close together. This matches with the calculations from the model because when a solar panel is tilted at an angle of 35°, it produces only 1.39 kWh/m² (or 0.22%) less per year than one tilted at 40°.

When the data from the mathematical model was plotted in **Figure 3**, it correctly and accurately simulates diurnal patterns, but there is a clear gap between the model data and field data. This is due to the fact that an empirical formula was used to calculate irradiance. The model should be calibrated by a coefficient so that the formula can account for atmospheric conditions that vary spatially and temporally. This would yield a more accurate value for the amount of annual energy produced. Ultimately, the gap between the model and the field data highlights that there are many other atmospheric factors to consider and a more complex model must be used to account for the variability in the atmospheric conditions.

After analyzing the data and comparing **Figure 4** and **Figure 5**, the daily field data seem to follow a similar pattern as the modeled annual solar energy data, with the exception of two outliers (tilt angles at 30° and 60°). The cause of the two outliers might be the result of individual solar panels having slightly lower or higher efficiencies ranging anywhere from 15% to 16% (17). The 30° panel having a higher efficiency than the norm and the 60° panel having a lower efficiency than the norm would result in more or less energy produced, respectively.

The data shown in **Figure 2** were taken on sunny days with no clouds and were set outside during sunrise and sunset. Data was recorded on cloudy days, as well; however, when the data analysis was performed, there were many inconsistent spikes in the data and no clear optimum tilt angle could be identified. When there are clouds, the sunlight scatters and reaches the solar panel from many different angles resulting in inconsistent data. For example, cumulus clouds in the sky result in large fluctuations in the amount of energy produced and the solar irradiance significantly decreases (12). In the future, the cloud impact should be analyzed because it has a large impact on solar panels.

In conclusion, knowing that the optimum tilt angle should be set to the latitude of the location, consumers can easily increase the annual efficiency of the solar panels. Although this tilt angle does not give the highest efficiency every month, this tilt angle will maximize the annual energy produced. According to **Figure 5**, when the energy produced by the 0° solar panel tilt angle (223 kWh/m²) is compared to the energy produced by the 40° solar panel tilt angle (287 kWh/m²), the percent difference is 28.69%. When this amount of energy saved is applied to the average roof area available for solar panels (45 m²) (12), then the annual increase in energy is around 2880 kWh ((287-223) kWh/m² × 45 m²).

In the future, field data at different locations and different dates should be recorded to further confirm the hypothesis. In addition, the true efficiency of each individual solar panel should be calculated to investigate whether each solar panel has the same efficiency. Lastly, a more complicated solar irradiance model should be implemented to account for the variable atmospheric conditions.

Materials and Methods

Modeling Work

The daily energy was an aggregate of hourly energy and the annual energy was calculated by adding the values of daily energy together. The hourly energy was calculated with the following formula:

$$\text{Energy} = \text{Irradiance} \times \text{Efficiency} \times \cos(\theta_s) \times t$$

Solar Irradiance: According to Kevin Addison from NASA, solar irradiance is defined as "the output of light energy from the entire disk of the Sun, measured at the Earth" (14). When sunlight passes through the Earth's atmosphere, a portion of the sun's rays is absorbed by the atmosphere. As shown in **Figure 7**, when the sun is hitting the Earth at an angle, it has to travel through more atmosphere than when the sun is directly overhead (15).

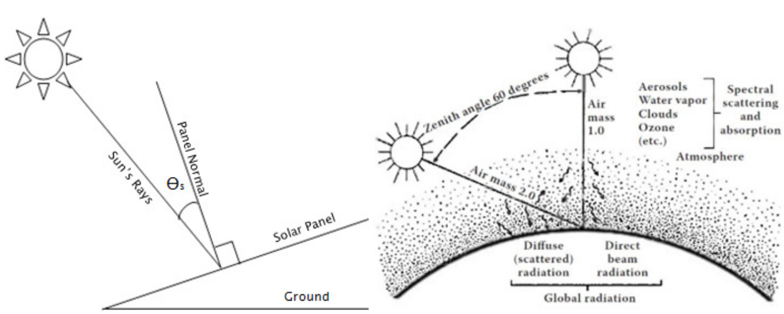

Figure 7. Diagram of Angle Between Sun's Rays and Panel Normal and Diagram of Air Mass. When the sun is at a solar zenith angle of 60°, it must travel through a longer distance of atmosphere. Due to the larger air mass, when the sun is at a larger zenith angle, it will experience more attenuation (absorption). In the morning when the sun is at an angle, the solar irradiance is low. Adapted from ref. 19.

This means that in the morning and the afternoon, not only is the intensity of the sun's rays lower, but there is also higher attenuation (absorption) in the atmosphere. This is modeled by the air mass (AM) in the following equation:

$$\text{Solar Irradiance} = 1.353 \times 0.7^{(AM^{0.678})}\ \text{kW/m}^2 \quad (10)$$

This formula for solar irradiance is a simplified calculation because the atmospheric conditions vary based on location and date. The value of 0.678 is an empirical fit to account for the varying atmospheric conditions and this value will vary based on location and date.

Air Mass = 1 / cos(Sun Zenith) (16)
Efficiency: 15 - 16% (17)
Time: 1 hour (3600 seconds)

$$\cos(\theta_s) = \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\cos(\omega)$$

where θ_s is the angle between sun's rays and normal of the solar panel; φ = Sun Zenith (calculated each hour, dependent on location) (angle between sun's position and directly overhead); δ = Tilt Angle (of solar panel relative to the ground); and ω = Sun Azimuth - Tilt Angle [Sun Azimuth (angle along the horizon 180° = South)].

Figure 5 displays the results of this model for annual energy output. **Figure 3** displays the results of this model for hourly energy on February 28th.

Field Measurements

The purpose of the field measurements is to see if the model is correct in that the latitude is the optimum tilt angle. Ten small solar panels, all tilted with different angles and facing south, were used. Using the breadboard, ten 20Ω resistors (one for each solar panel) were placed in the circuit to help calculate the energy the solar panel generated. A CR5000 datalogger was connected to each of the solar panels and programmed to measure the voltage across the resistor every 5 minutes from sunrise (7 am) to sunset (8 pm). See **Figure 8** for the complete set up. To prevent damage to the solar panels, they were cleaned after a day of experimentation and were left inside when they were not taking measurements. The following formula was used to calculate hourly energy per unit area.

$$E/A = (U^2 \times t)/(R * A)$$
$$[E/A] = kWh/m^2$$

where E is energy in kWh; A = Area of the solar panel = .005612 m^2; (17); U = voltage (average voltage in 1 hour according to datalogger); t = time (1 hour); R = Resistance (value of the resistor used was 20 ohms)

The formula for hourly energy per unit area was derived the following way: (18)

1. Power = (Voltage * Current), and Power = (Energy / Time), therefore Energy = (Voltage * Current * Time).
2. Substituting Current = (Voltage / Resistance) (18) yields $E = (U^2 \times t) / R$.
3. Divide by area, to get hourly energy per unit area: $E/A = (U^2 \times t) / (R * A)$

Energy per unit area was used so that the energy value does not depend on the size of the solar panel, because produced energy is directly proportional to the area of the solar panel. The values obtained from this formula were used in **Figure 2** to show energy produced by the solar panels on February 28th and March 1st.

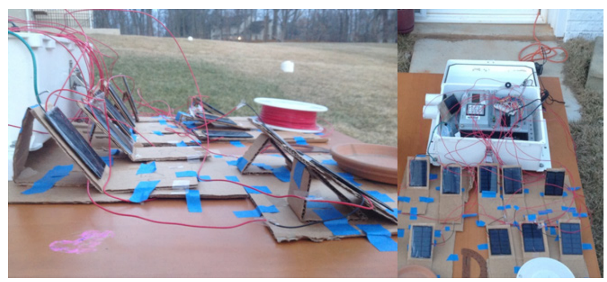

Figure 8. Solar Panel and Data Logger Setup. The solar cells, all set to different angles, were set outside from dawn to dusk and the voltage produced across a resistor was logged by a data logger every 5 minutes. The set up of the 10 multi-angled solar panels is pictured to demonstrate that each solar panel is set to a specific angle (0°, 15°, 30°, 35°, 40°, 45°, 50°, 55°, 60°, 75°) based on a triangular cardboard structure.

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